UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0718395175

MATHEMATICS (SYLLABUS D) **4024/22**

Paper 2 **October/November 2011**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Geometrical instruments
Electronic calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Section A
Answer **all** questions.

Section B
Answer any **four** questions.

If working is needed for any question it must be shown in the space below that question.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

For Examiner's Use

This document consists of **24** printed pages.

DC (CW/DJ) 35498/3

UNIVERSITY *of* CAMBRIDGE International Examinations

[Turn over

Section A [52 marks]

Answer **all** questions in this section.

1 **(a)** $A = h(4m + h)$

Express m in terms of A and h.

Answer .... [3]

(b) Factorise completely $3ax + 5bx - 6ay - 10by$.

Answer .... [2]

(c) Solve the equation $\dfrac{5x-1}{9} = \dfrac{9}{5x-1}$.

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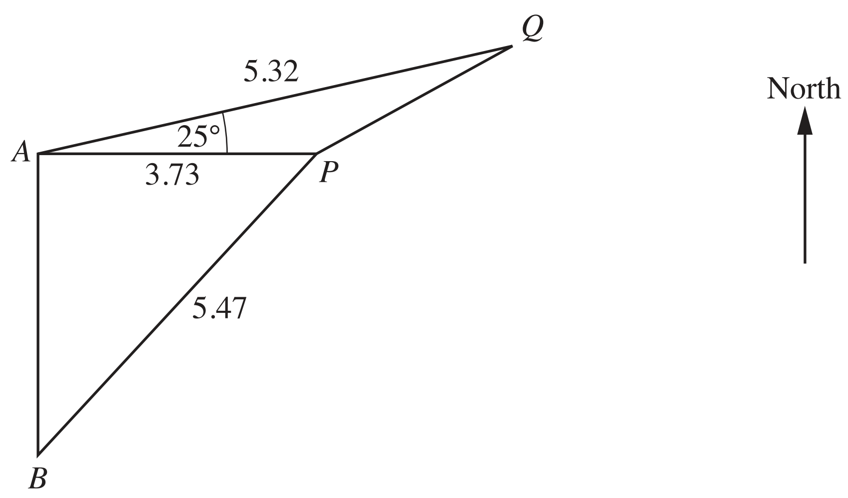
Answer $x =$ .... or .... [3]

2



The diagram shows four points, A, B, P and Q, at sea.
B is due South of A and P is due East of A.
$AP = 3.73$ km, $BP = 5.47$ km, $AQ = 5.32$ km and $P\hat{A}Q = 25°$.

(a) Calculate $A\hat{B}P$.

Answer .... [2]

(b) Calculate PQ.

Answer ....km [4]

(c) A boat sailed in a straight line from Q to A.

(i) Find the bearing of A from Q.

Answer .... [1]

(ii) A lighthouse is situated at A.
The top of the lighthouse is 30 m above sea level.
Calculate the angle of depression of the boat from the top of the lighthouse when the boat is 100 m from A.

Answer .... [2]

3 (a)

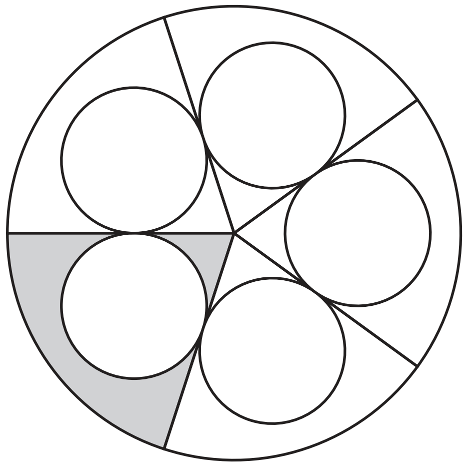


Diagram I

Diagram I shows one large circle and five identical small circles.
Each of the five radii shown is a tangent to two of the small circles.

(i) Describe the symmetry of the diagram.

Answer .... [1]

(ii) The radius of the large circle is R centimetres and the radius of each small circle is r centimetres.
Each small circle is equal in area to the shaded region.

Find $R^2 : r^2$.

Answer .... : ....[3]

(b)

Diagram II

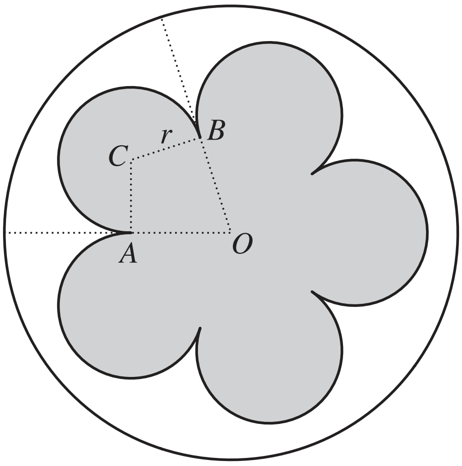


Diagram II shows the same large circle and arcs of the same small circles as in **Diagram I**.
C is the centre of one of the small circles.
This circle touches the adjacent circles at A and B.
O is the centre of the large circle.

(i) Show that reflex $A\hat{C}B = 252°$.

[2]

(ii) The perimeter of the shaded region is $k\pi r$ centimetres.

Calculate the value of k.

Answer .... [2]

4 **(a)** A shopkeeper buys some plates from a manufacturer for $10 each.

(i) **(a)** The shopkeeper sells a plate for $12.

Calculate the percentage profit.

Answer ....% [1]

(b) The shopkeeper buys 240 plates and sells 180 at $12 each.
The rest were sold to a café for a total of $540.

Calculate the percentage discount given to the café.

Answer ....% [2]

(ii) The manufacturer made a profit of 60% when he sold each plate for $10.

Calculate the cost of manufacturing each plate.

Answer $.................................... [2]

(b) Another shopkeeper bought 100 pans at \$5 each.
He sold 63 at \$6 each and x at \$4 each.
He did not sell all the pans nor enough to make an overall profit.

(i) Form an inequality in x.

Answer .... [1]

(ii) Hence find the greatest possible number of pans that were sold.

Answer .... [2]

(c) One day, the rate of exchange between American dollars (\$) and British pounds (£) was \$1.45 = £1.

(i) Alan changed £300 into dollars.

Calculate how many dollars he received.

Answer \$.................................... [1]

(ii) On the same day, the rate of exchange between South African rands (R) and pounds was R10.44 = £1.

Calculate the number of rands received in exchange for one dollar.

Answer R.................................... [2]

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5 **(a)** $\begin{pmatrix} 3 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ 11 \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$.

Find x and y.

Answer x = ...

y = ... [2]

(b) **(i)** The transformation A is represented by the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Find, in terms of a, b, c and d as appropriate,

(a) the image of (1, 0) under the transformation A,

Answer (........... , ...) [1]

(b) the image of (0, 1) under the transformation A.

Answer (........... , ...) [1]

(ii) The transformation B maps (1, 0) onto (1, 3) and (0, 1) onto (–3, –2).

Write down the matrix that represents transformation B.

Answer $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

(iii) Describe fully the transformation given by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Answer ....

.... [2]

6 You may use the graph paper on the next page to help answer this question.

The point A is (0, 7), and the point B is (6, 9).

(a) Express $\overrightarrow{AB}$ as a column vector.

Answer [1]

(b) Find the gradient of AB.

Answer .... [1]

(c) The equation of the line AB is $x + Py + Q = 0$.

Find P and Q.

Answer P = ...

Q = ... [2]

(d) The point C is (12, 2).

(i) Given that C is the midpoint of BM, find the coordinates of M.

Answer (........... , ...) [1]

(ii) Calculate AC.

Answer .... units [1]

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(iii) The point D lies on the line AB.
The line CD is parallel to the y-axis.

(a) Find the coordinates of D.

Answer (........... , ...) [2]

(b) Express $\overrightarrow{AD}$ in terms of $\overrightarrow{AB}$.

Answer $\overrightarrow{AD}$ = ... [1]

Section B [48 marks]

Answer **four** questions in this section.

Each question in this section carries 12 marks.

7

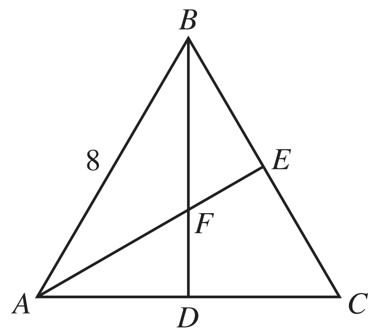


Diagram I

In **Diagram I**, ABC is an equilateral triangle of side 8 cm.
D and E are the midpoints of AC and BC respectively.
BD and AE intersect at F.

(a) **(i)** Find the area of triangle ABC.

Answer ....cm^2 [2]

(ii) Show that $A\hat{F}B = 120°$.

Answer ....

.... [1]

(iii) Calculate AF.

Answer .... cm [2]

(b) **[The volume of a pyramid = $\frac{1}{3}$ × base area × height]**

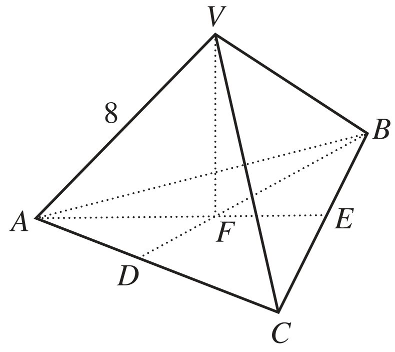


Diagram II

The equilateral triangle of side 8 cm in **Diagram I** forms the base of the triangular pyramid *VABC* in **Diagram II**.
The vertex *V* is vertically above *F*.
$VA = VB = VC = 8$ cm.

(i) Calculate the surface area of the pyramid.

Answer ....cm^2 [1]

(ii) Calculate the volume of the pyramid.

Answer ....cm^3 [3]

(c) A pyramid *P* is geometrically similar to *VABC* and its volume is $\frac{1}{64}$ of the volume of *VABC*.

(i) Find the length of an edge of *P*.

Answer .... cm [2]

(ii) A pyramid that is identical to *P* is removed from each of the four vertices of *VABC*.

State the number of faces of the new solid.

Answer .... [1]

8 Two companies, A and B, were started 10 years ago.
Initial investments of $25 or multiples of $25 could be made when Company A started business.

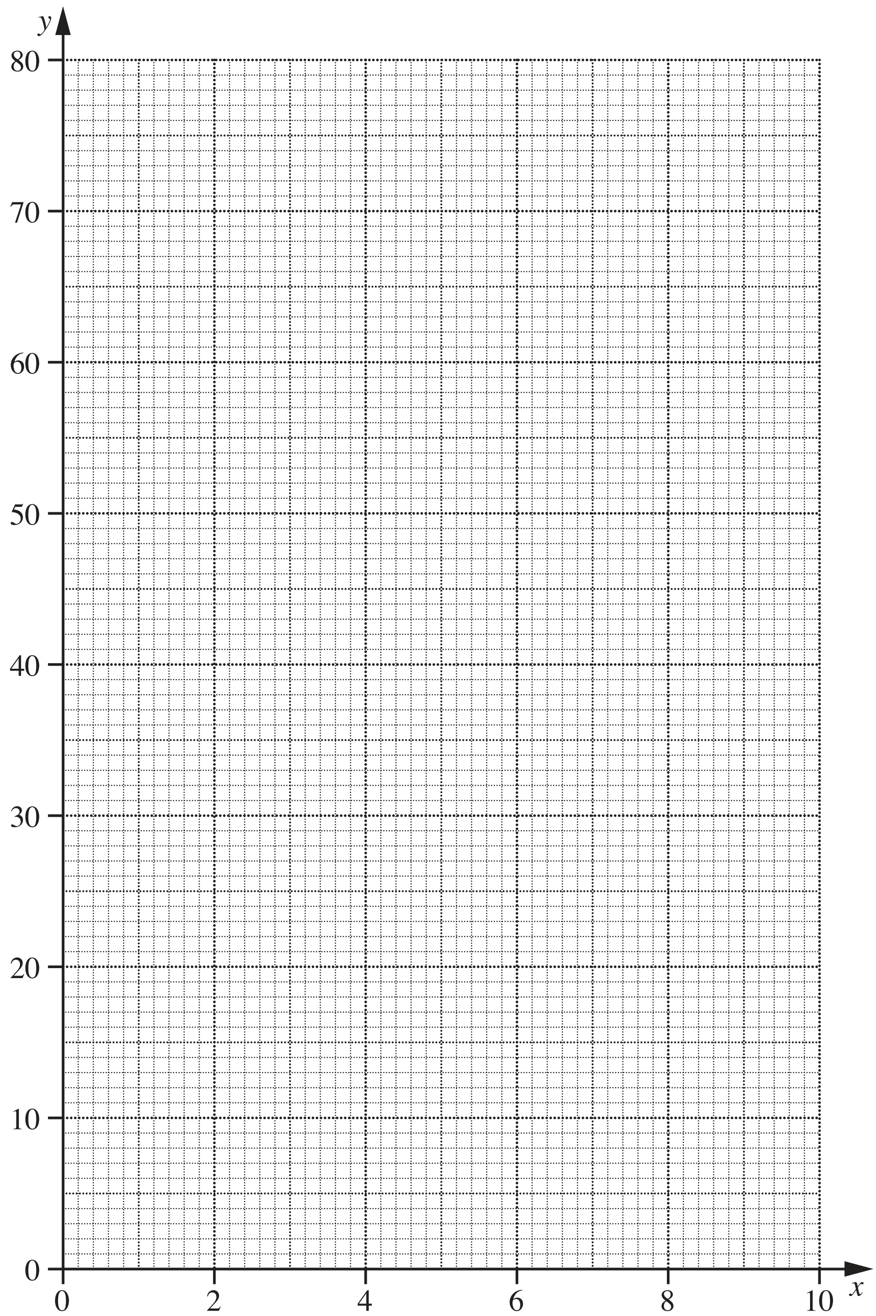
(a) The table shows the value of an initial investment of $25 at the end of each of the next 10 years.

Number of years (x)	0	1	2	3	4	5	6	7	8	9	10
Value in dollars (y)	25	28	31	35	39	44	49	55	62	69	78

(i) Calculate the value of an initial investment of $500 after 8 years.

Answer $.................................... [1]

(ii) On the grid, plot the points given in the table and join them with a smooth curve.



[2]

(iii) Using your graph, find x when the value of an initial investment of \$100 had increased to \$168.

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Answer .... [1]

(b) An initial investment of \$25 was made when company B started business.
The value, y dollars, after x years, is given by the equation $y = 3.75x + 25$.

(i) Calculate the value of an initial investment of \$500 after 8 years.

Answer \$.................................... [1]

(ii) On the grid, draw the graph of $y = 3.75x + 25$. [2]

(c) Using your graphs, find the value of x when an initial investment of \$25 had increased to the same value in each company.

Answer .... [1]

(d) **(i)** By drawing a tangent to the graph representing an investment in company A, find the rate of increase of this investment when $x = 7$.

Answer .... [2]

(ii) **State** the rate of increase of an investment in company B.

Answer .... [1]

(iii) By drawing another tangent to the graph representing an investment in company A, find the value of x when the rates of increase of investments in each company were the same.

Answer .... [1]

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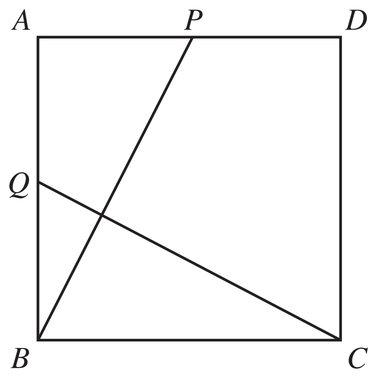
Diagram I



In **Diagram I**, $ABCD$ is a square.
P and Q are the midpoints of AD and AB respectively.

(a) Show that triangles APB and BQC are congruent.

Answer ....

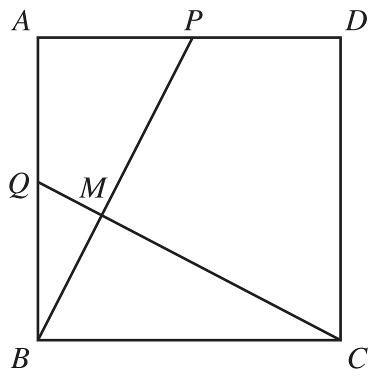
....

....

.... [3]

(b)

Diagram II



In **Diagram II**, QC and PB intersect at M.

Show that $B\hat{M}C = 90°$.
State your reasons clearly.

Answer ....

....

.... [2]

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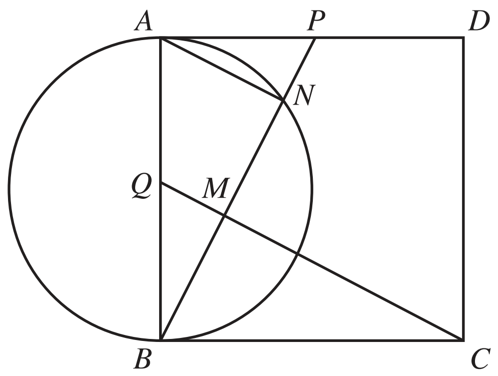
(c)

Diagram III



In **Diagram III**, the circle centre Q has diameter AB.
The circle intersects BP at N.

(i) State the reason why $A\hat{N}B = 90°$.

Answer .... [1]

(ii) Triangle BMQ is mapped onto triangle BNA by an enlargement.
Write down the centre and scale factor of the enlargement.

Answer ....

.... [1]

(iii) Given that $QM = 3$ cm,

(a) find AN,

Answer .... cm [1]

(b) show that $MN = 6$ cm,

Answer ....

.... [1]

(c) find MC,

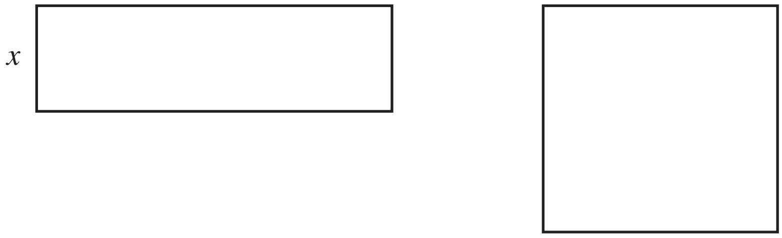
Answer .... cm [1]

(d) find the area of triangle APB.

Answer ....cm^2 [2]

[Turn over

10



A piece of wire, 28 cm in length, is cut into two parts.
One part is used to make a rectangle and the other a square.

The length of the rectangle is three times its width.
The width of the rectangle is x centimetres.

(a) **(i)** Write down an expression, in terms of x, for the length of the rectangle.

Answer .... cm [1]

(ii) Find, and simplify, an expression, in terms of x, for the length of a side of the square.

Answer .... cm [2]

(b) It is given that the area of the rectangle is equal to the area of the square.

(i) Form an equation in x and show that it reduces to $x^2 - 28x + 49 = 0$.

[2]

(ii) Solve the equation $x^2 - 28x + 49 = 0$, giving each solution correct to 3 significant figures.

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Answer $x =$ .... or .... [4]

(iii) Which solution represents the width of the rectangle?
Give a reason for your answer.

Answer The width of the rectangle is ....cm because ....

.... [2]

(iv) Calculate the area of the square.

Answer ....cm^2 [1]

11 **(a)** A sports club has 120 members.
The cumulative frequency table for their ages is shown below.

Age (x years)	$x \leqslant 5$	$x \leqslant 15$	$x \leqslant 25$	$x \leqslant 35$	$x \leqslant 45$	$x \leqslant 55$	$x \leqslant 65$
Cumulative frequency	0	12	30	60	96	114	120

(i) On the grid on the next page
draw a horizontal x-axis for $0 \leqslant x \leqslant 70$, using a scale of 2 cm to represent 10 years and a vertical axis from 0 to 120, using a scale of 2 cm to represent 20 members.

On your axes draw a smooth cumulative frequency curve to illustrate the information in the table. [3]

(ii) Find the upper quartile age.

Answer ....years [1]

(iii) Find the interquartile range of the ages.

Answer ....years [1]

(iv) Members who are not more than 15, and members who are over 50, pay reduced fees.
Use your graph to find an estimate of the number of members who pay reduced fees.

Answer .... [1]

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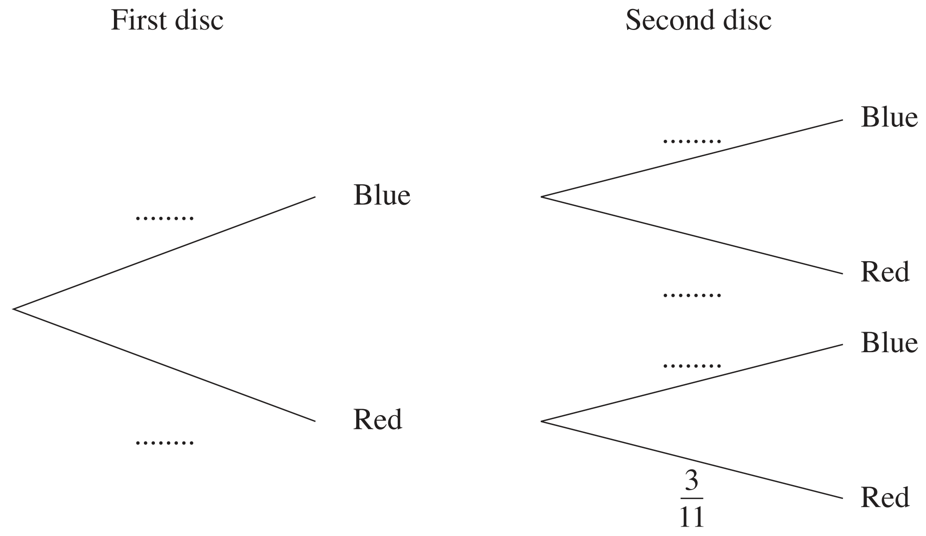
Turn over for the rest of this question.

(b) A bag contains 12 discs.
There are 8 blue and 4 red discs.

A disc is picked out at random and not replaced.
A second disc is then picked out at random and not replaced.

The tree diagram below shows the possible outcomes and one of their probabilities.



(i) Complete the tree diagram. [2]

(ii) Expressing each of your answers as a fraction **in its lowest terms**, calculate the probability that

(a) both discs are red,

Answer .... [1]

(b) at least one disc is blue.

Answer .... [2]

(iii) A third disc is picked out at random.
Calculate the probability that all three discs are red.

Answer .... [1]